

***Assessment of the Quality of Geodetic Networks
Using Fuzzy Logic***

Gintcho Kostov

**Vienna University of Technology
Institute of Geodesy and Geophysics
Research Unit Advanced Geodesy**

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Abstract

One of the main tasks when dealing with the adjustment of geodetic networks is to estimate their accuracy and condition. Generally two distinct ways exist for the assessment of the quality of geodetic networks: either a pure mathematical approach with exact criteria or fuzzy logic, where linguistic terms are used. In order to produce reliable results in the case of many parameters and conditions, the networks under investigation are assessed with fuzzy logic. For this study a software for processing the results of the adjustment used as input variables for the fuzzy logic application has been developed. Its final task is computing rating values in the interval $[0,1]$ showing the quality of the system, i.e. in our case the adjusted geodetic network. Conclusions are made, taking in mind the computed rating values for various geodetic networks.

keywords: network, measurement, assessment, surveying, condition, accuracy, quality, adjustment, geodesy, geodetic, мрежа, измерване, оценка, изследване, обусловеност, точност, качество, изравнение, геодезия, геодезическа.

1. Introduction

The classical way for assessing geodetic networks is based on various criteria (described in the next chapter), derived within the adjustment procedure. The human expert should decide which system amongst others has the best quality, based on each criterion. The technical problem is that some of the mathematical criteria known from literature are derived in parallel with the processing, whereas for others additional computations are necessary. Due to this reason and because of the complexity of the problems in some occasions the geodesists use only a few criteria, neglecting the others. However, for a complete network analysis all available criteria for accuracy and condition should be considered in order to get reliable results.

Generally, using fuzzy logic is a way to assess a system, which has a number of parameters varying between large and small values. It is in particular useful when no exact deterministic model exists for the problem, but rules can be found by human experts, solving the problem by their experience. In this case, fuzzy logic is an alternative for assessing both condition and accuracy of geodetic networks. One of the important questions in geodesy is the choice of appropriate weights. In some of the articles the application of fuzzy logic in geodesy is treated only under the assumption of equal weights. As it will be shown here, in

fuzzy logic weights have their significance, too. An application has been developed, which reads the variables, values, rules and after finishing all processes calculates the rating value for the given system.

2. Fuzzy logic

2.1 Some general information

The general concept of fuzzy logic is described e.g. by Wieser (2001). A fuzzy set consists of pairs of values of a given variable X and relevant values of a characteristic membership function $\chi(X)$, often abbreviated as MF. The value of MF is in the range of $[0,1]$, indicating the so-called degree of membership. Value “0” indicates non-membership, while “1” indicates full membership. The difference to the classical concept of set theory is that with fuzzy sets, also degrees of membership between 0 and 1 are possible.

The main idea of fuzzy variables is linguistic: e.g. “small”, “not big”, “rather big”, “big”. For example, one can treat a number around 0.1 as “small”, a number around 0.6 as “not big” and a number around 0.9 as “big”. The general scheme for the so-called fuzzy controller is:

Input>Fuzzification>Inference>Defuzzification

In the first part certain values are entered, which are then fuzzified, this means with the relevant MF they get their value for the degree of membership. In the inference part the weights are given and the relevant operator (“and”, “or”) is applied. The final part called defuzzification is used to obtain a crisp value for the rating. There are several methods to perform this final part of the calculation, but the most appropriate and commonly used one is the *centroid* method of defuzzification.

Fuzzy logic is a simple way to “plot” an input region into an output region. An example: *if I know how good was the lunch at your favourite restaurant today, I could tell you what is the tip you should give.*

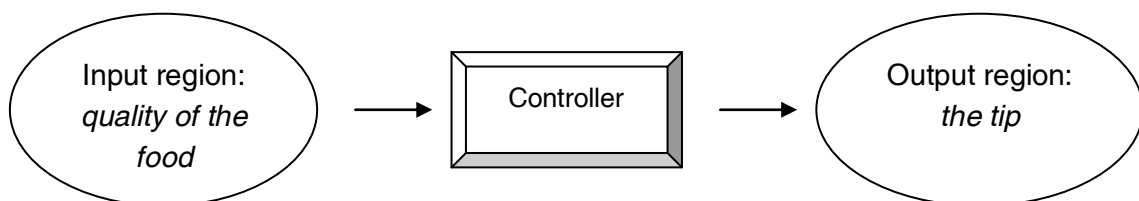


Figure 1: Graphical example for a fuzzy logic system

The common structure of a rule is: input variables, resulting MF, weight, logical operator (“and (1)”, “or (2)”). For example, if the user defines a, b, and c as input variables, the rules may look in the following way:

a	b	c	resulting MF	weight	logical operator
-1	0	0	1	0,4	2
0	-1	0	1	0,85	2
0	0	-1	1	0,9	2
1	0	0	2	0,4	2
0	1	0	2	0,85	2
0	0	1	2	0,9	2

Figure 1a: Example and description for six rules

In natural language the last rule has the following meaning: If (c is small) then the network is *goodnetwork*, (weight).

After the calculation, the user will get a value between [0,1], telling the quality of the system. In this particular case, for example if we derive a rating of 0.85 we can say that the system is *goodnetwork*. In case we get 0.95 it can be said that the system is also *goodnetwork*, but much better. But if the rating is 0.25, the system is considered as *badnetwork*. One can assess any kind of system, with given input variables and user-constructed rules.

2.2 Applications of fuzzy logic in geodesy

The possible use of fuzzy logic for various geodetic tasks (GPS, data processing, landslide monitoring, etc.) has been described in several publications, e.g. by Heine (2001), Kutterer (2001), Leinen (2001), Wieser (2001), Haberler (2003), Wieser (2003). Many more applications of this logic might be possible in geodesy, depending on the specific needs. One new additional possible usage for the assessment of the quality of geodetic networks will be given in this paper.

3. Application of fuzzy logic for assessing geodetic networks

3.1 Mathematical Basics

A brief explanation of the used symbols in the article will be given here:

N - Normal equation matrix;

Q - Co-factor matrix;

cond(.) - Condition number of a matrix;

Tr() - Trace of a matrix;

Det() - Determinant of a matrix;

Mar - Mean arithmetic error for the whole network;

Msq - Mean quadratic error for the whole network;

m_p - Mean error in the position of a new point;

nn - Number of the new-determined points;

n - Dimension of a matrix;

a_{ij} - Element of a matrix;

λ_i - Eigenvalue of the Q matrix;

B_i - Hyper ellipsoid semi-axis.

In this research the following criteria for assessing the quality of geodetic networks are used, all of them based on parameters obtained by a standard least-squares fit:

Sum Tr(Q) of the diagonal elements of the co-factor matrix of parameters:

$$\text{Tr}(Q) = \sum_{i=1}^n Q(i, i) \quad (1)$$

Determinant Det(Q) of the inverse matrix Q of the normal equation matrix N :

$$\text{Det}(Q) = \prod_{i=1}^n \lambda_i \quad (2)$$

Mean arithmetic error Mar for the whole network:

$$\text{Mar} = \frac{\sum m_{p_i}}{nn} \quad (3)$$

Mean quadratic error Msq for the whole network:

$$\text{Msq} = \sqrt{\frac{\sum m_{p_i}^2}{nn}} \quad (4)$$

Semi-axes B_i of the ellipse, respectively hyper ellipsoid of errors:

$$B_i = \mu \sqrt{\lambda_i} \quad (5)$$

where μ is the RMS for weight 1. This criterion is both used locally, i.e. for each new point, and globally, i.e. for the whole network.

The product:

$$\varepsilon \cdot \text{cond}(N) \quad (6)$$

described by Konstantinov and Vulchanov (1987), where $\varepsilon = 2.2204e-016$ and N is the normal equation matrix. It is known, that if $\varepsilon \cdot \text{cond}(N) \ll 1$, the matrix N is well conditioned. With $\text{cond}(N) = \|N\| \|N^{-1}\|$ the condition number of the matrix under investigation is denoted.

We also use the criteria N_{number} and M_{number} , given in Faddeev and Faddeeva (1963), using the design matrix A :

$$N_{\text{number}} = \frac{1}{n} N(N_A) N(N_A^{-1}); \quad M_{\text{number}} = \frac{1}{n} M(N_A) M(N_A^{-1}); \quad (7)$$

$$N(N_A) = \sqrt{\text{Tr}(N_A^T N_A)}, \quad M(N_A) = n \cdot \max_{ij} |a_{ij}| \quad (8)$$

It must be noted, that in order to avoid confusion, only in (7) and (8) the normal equation matrix is denoted with N_A .

P number:

$$P = \frac{\max |\lambda_i|}{\min |\lambda_i|} \quad (9)$$

All criteria are derived from the adjustment process of the geodetic networks. Later on they will be used as input variables in fuzzy logic.

3.2 Software information

In order to customize, ease and mechanize the calculation of the rating values, especially for geodetic purposes an application has been developed in the OS Windows XP environment. The actual aim of the application is to assess the quality of a given geodetic network (measured by triangulation and trilateration), based on the computed rating with appropriately assigned weights for each input variable.

The main characteristics and capabilities of the application are:

- a) It is possible to enter the names of the variables and their values by hand or from file.
- b) A check for inconsistency of the input parameters is performed.
- c) Appropriate weights are calculated for each variable, as will be demonstrated in section 3.3.
- d) Construction can easily be performed, editing, addition and removal of the rules, more than one simultaneously.
- e) A check for blunders in the rules is also done.
- f) It is possible to perform sequential computations after changing the necessary parameters.
- g) The last computations are optionally saved for further referencing.

- h) The rating of any given system can be evaluated with minimum effort.
- i) For the current needs the case is considered with two linguistic terms, describing the output i.e. *badnetwork* and *goodnetwork* (see section 3.4).

The input and editing of the rules is organized in a memo box, which allows the user to easily change the parameters of the rules, including the weights.

In case the user enters the data from the adjustment *by hand*, the following is required:

- names of input variables;
- their values;
- type of the input membership function - either ZMF (function, which has a shape as the letter “Z”, figure 2);
- or SMF (function, which has a shape as the letter “S”, figure 3), the default is set to ZMF.



Figure 2: View of the ZMF function Figure 3: View of the SMF function

- minimum and maximum input range;
- required parameters for each input function, corresponding to each variable;
- names of the linguistic terms, two are necessary for current purposes;
- values of the output parameters, there are default set values;
- rules, simply with $-1, 0, 1$;
- weight for each rule, one by default;
- resulting variable (*badnetwork* or *goodnetwork*);
- logical operator in the rule: or/and.

3.3 Weights of the rules

The weights p_i can be calculated using the equation:

$$p_i = \frac{\text{MinParameter}}{\text{Value}_i}, \text{ for Value}_i \neq 0 \quad (10)$$

where *MinParameter* is the smallest parameter of the relevant membership function used for the calculations. The values of the weights will be between $[0,1]$. Thus, every variable will get its most-appropriate weight and the system (in our case the network) will be assessed in a strict way.

In our investigation the weights were determined according to one of three different options, in order to compare the results:

1. Each rule has the weight 1;
2. Each rule has a weight calculated according to equation (10);
3. Each rule, consisting of different variables has a weight determined by the human expert.

3.4 Assessment of geodetic networks

The following studies were done within this research:

1. Construction of the rules with a single variable according to section 3.3.
2. Including only certain variables in the rules in order to investigate specific characteristics of a given network.
3. Investigating the quality of specific small networks and also of a big network based on sets of variables and the geographical positions of certain fixed points within the geodetic network.

In order to decide which network variant from a given set has a good quality in terms of accuracy and condition, the values of the relevant criteria were determined according to section 3.1. The key moment for the user is to define the rules, based on human experience, with appropriate weights.

For example:

If $Tr(Q)$ is small then system is goodnetwork -> high weight,

If $Tr(Q)$ is small or $Det(Q)$ is small then system is goodnetwork -> medium weight,

If $Tr(Q)$ is big then system is badnetwork -> small weight,

where “*badnetwork*” and “*goodnetwork*” are the linguistic terms, describing the quality of the system. From the input data (values of variables, rules, etc.), the application computes the rating value.

Experiments were performed with geodetic networks having various geometry and different number of fixed points which all are part of a big network (figure 4) – along the meridian (figure 5), along the parallel (figure 6), and in a square area (figure 7). The tests were done in order to explore certain characteristics of the networks and also to detect eventual differences of the rating values depending on certain input data. In some of the variants coordinates, treated as measurements, were added to the usual angular and distance measurements to improve the quality.

3.5 Results and remarks

The comparison of the rating values using either equal weights or appropriately assigned weights is given in figure 8. Detailed descriptions of the various solutions, the variables and the rating values are given in table 1. For network M3 (fixed points 134, 153 from the network along the meridian with additional coordinate measurements) and also for network M4 (fixed points 99, 100 from the network along the meridian) the rating is higher in comparison with network M1 (fixed points 49, 50 from the network along the meridian) and network M2 (fixed points 134, 153 from the network along the meridian). The difference between the rating values with equal or appropriately assigned weights in network M1 and M2 is due to the fact that they were not suitably chosen. Similar results are valid for networks P1 to P5. A high value is obtained when coordinate measurements were added, e.g. in P4, compared to P3, or when the fixed points are positioned in the opposite part of the network, see P2. From estimating the network with a square area shape it can be concluded, that the highest rating is derived when the fixed points are positioned in the central part of the network. As a result it can be said that there is a slight change in the rating values when using different weights instead of equal weights.

The quality of a big classical network (figure 4) was investigated within nine variants with various locations of the fixed points (table 2 and figure 9). Major differences of the rating values can be observed in network variant N6. The reason for this low rating are mainly the large values of the used criteria and the position of the fixed points on the western edge of the network.

The network with a square area shape (figure 7) was under investigation with variables and rating values summarized in table 3 and figure 10. For this case six input variables were used. The reason for the low rating of networks 1, 5, 7, 10 might be due to the small number of fixed points or their specific position. High rating between 0.74 and 0.79 was obtained when coordinates were added as “measurements” for improving the quality of the network. It should be noted that there is no significant difference in the rating when the coordinate measurements were increased even to 8 (network 12-15). As a result it can be said that almost the same rating was obtained when no coordinate measurements were added, but the fixed points are at the periphery of the network according to variant 11. Thus, the quality of a network can be assessed with fuzzy logic to decide about the number of coordinates to be added.

It should be noted that the rating value for an assessment with fuzzy logic might vary, if the input parameters of the functions are changed by entering additional

results from the adjustment. In order to avoid such a casual change, all basic variables should be entered at once. Fuzzy logic should produce similar results as those derived by the human expert, as the rules are created from the user. However, fuzzy logic is more reliable than the human expert, when there are a lot of variables and conditions.

4. Conclusions

From the calculations and experiments done in this study, it can be summarized:

When performing an assessment of a network, one should use all available variables and avoid construction of a sophisticated system of rules. In this case it is preferably that the rules are simple with one variable in each rule. It is proposed not to compose multiple variables-rules.

Weights should be determined precisely, according to section 3.3. The user should not determine the weights by her/himself. It is recommended that the human expert examines whether the rating corresponds to reality. The output generally depends on the value of each variable and/or of the logical operator when using rules with many variables. It is proposed to apply both classical and fuzzy methods in order to get a complete and reliable network analysis.

In this way a well-conditioned system can be quickly and easily chosen among others. Rating values derived with fuzzy logic could be used as an additional information for the geodesists.

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Used software:

Matlab fuzzy logic toolbox;

Vienna_fuzzy (Application developed for the needs of this research);

PhDSf (Application, subject to the PhD thesis of the author).

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Table 1
Rating values (with equal and appropriate weights) of the networks with five variables

Number of network	Kind of sub-network	Coordinates fixed for points	Criteria (variables)					Rating	
			Tr(Q)	Det(Q)	Max Hyper Ellipsoid Semiaxis [mm]	Eps*cond(N)	Mnumber	Rating with equal weights	Rating with different weights
M1	Meridian Net	49, 50	2,03E+04	3,82E+35	1,41E+03	1,77E-11	1,45E+05	0,50	0,36
M2		134, 153	1,95E+04	8,73E+35	1,37E+03	1,72E-11	1,35E+05	0,51	0,36
M3		134, 153 and added additional measurements	5,17E+03	1,14E+30	7,21E+02	4,99E-12	6,24E+04	0,68	0,74
M4		99, 100	4,96E+03	1,06E+35	5,53E+02	2,80E-12	2,80E+04	0,75	0,76
P1	Parallel net	100, 104, 112, 114	4,51E+03	1,31E+33	5,66E+02	2,65E-12	4,78E+04	0,74	0,76
P2		100, 104, 121, 2790	2,27E+03	3,31E+31	3,12E+02	1,14E-12	5,64E+03	0,80	0,80
P3		100, 104	4,26E+04	2,16E+40	1,85E+03	3,35E-11	3,31E+05	0,50	0,20
P4		100, 104 and added additional measurements	2,89E+03	3,45E+31	5,55E+02	2,14E-12	3,55E+04	0,75	0,77
P5		112, 114	8,83E+03	4,78E+40	6,07E+02	3,78E-12	5,45E+04	0,73	0,75
A1	Area net	98, 100	2,66E+04	1,07E+60	9,57E+02	9,12E-12	3,21E+04	0,52	0,58
A2		50, 149	2,23E+04	1,36E+61	9,16E+02	7,80E-12	3,88E+04	0,50	0,61
A3		50, 149 and added additional measurements	3,66E+03	1,09E+53	1,00E+03	1,20E-12	2,18E+04	0,57	0,69
A4		58, 59	6,76E+03	9,31E+60	4,03E+02	1,64E-12	8,63E+03	0,53	0,79
A5		60, 633	1,49E+04	1,68E+61	6,91E+02	4,66E-12	2,71E+04	0,50	0,72
A6		47, 632	2,08E+04	7,50E+60	8,90E+02	7,47E-12	4,10E+04	0,56	0,62

Figure 8
 Rating of the meridian network, parallel network and network with a square area
 shape (see Table 1)
 Calculated with $\text{Tr}(Q)$, $\text{Det}(Q)$, maximum hyper ellipsoid semi-axis,
 $\text{eps} \cdot \text{cond}(N)$, Mnumber.

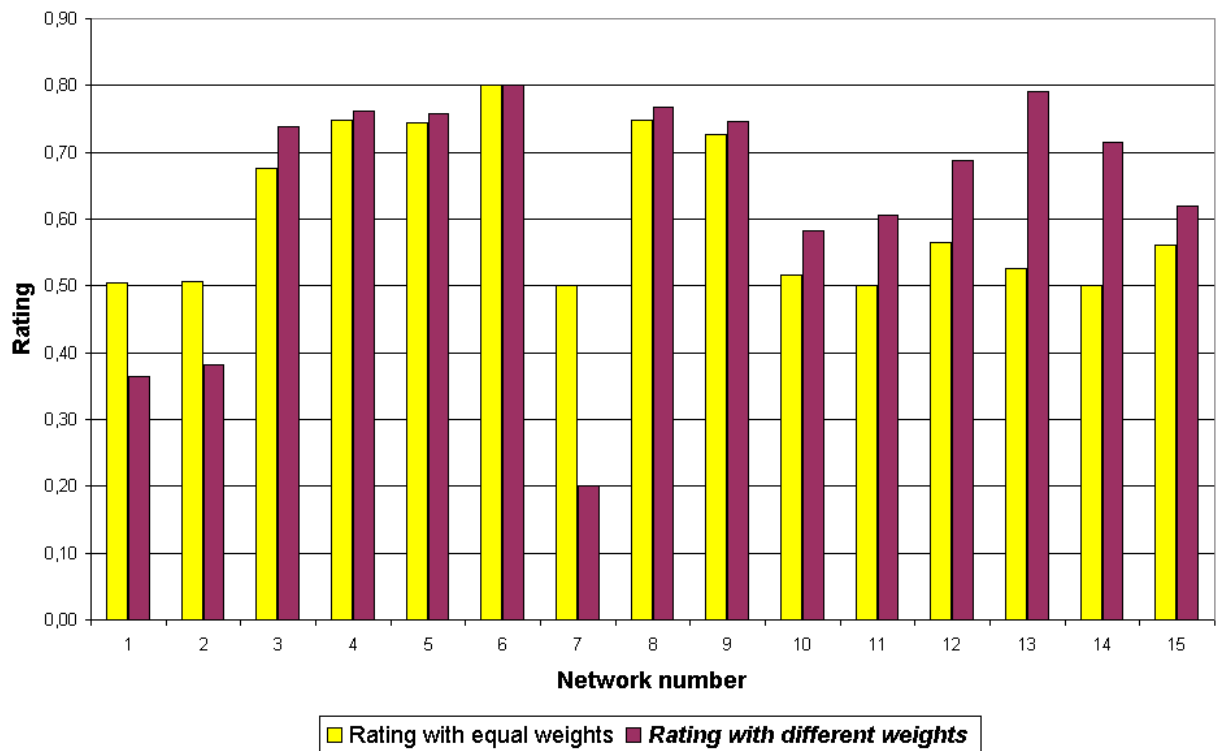


Table 2
 Input variables and rating values of the big network (Figure 4) with nine variants,
 using two sets of variables.

Number of variant	Fixed points	Criteria (variables)						Rating	
		Tr(Q)	Mnumber	Nnumber	eps*cond(N)	Msq	Mar	First set used variables: Tr(Q), Mnumber, Nnumber,	Second set used variables: Tr(Q), Mnumber, Nnumber, eps*cond(N), Msq, Mar
N1	154, 632	9058,5	1193759,2	1178,3	5,91E-11	117,4	100,5	0,80	0,78
N2	131, 1163 and added additional measurements I-st set	2809,7	1078695,7	306,4	1,54E-11	159,8	97,0	0,80	0,76
N3	131, 1163 and added additional measurements II-nd set	2784,7	1078700,2	304,8	1,53E-11	101,5	63,3	0,80	0,79
N4	131, 1163	6384,0	1078788,9	739,5	3,71E-11	99,1	82,1	0,80	0,79
N5	90, 112	7809,4	1125876,9	964,0	4,84E-11	109,0	90,0	0,80	0,79
N6	617, 1159	185305,3	4170793,6	32735,9	1,64E-09	531,1	503,3	0,20	0,20
N7	49-50 and added additional measurements	2790,6	1087574,8	303,3	1,52E-11	66,5	41,0	0,80	0,80
N8	45-633	15946,0	1345421,4	2367,8	1,19E-10	155,8	137,7	0,79	0,76
N9	100, 104	10642,7	1126818,5	1467,2	7,36E-11	127,3	110,4	0,80	0,78

Figure 9

Values of the input variables and fuzzy logic rating for investigating the condition of the big network with nine variants.

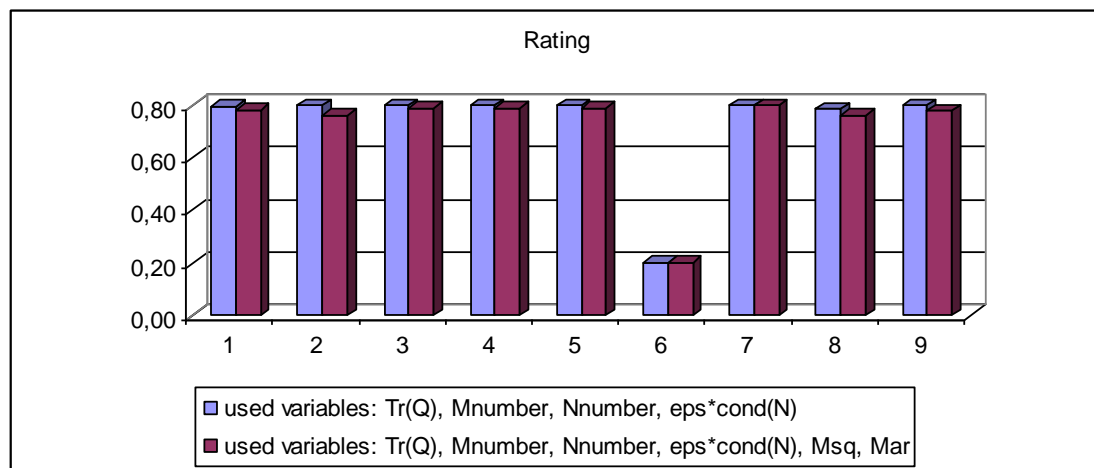
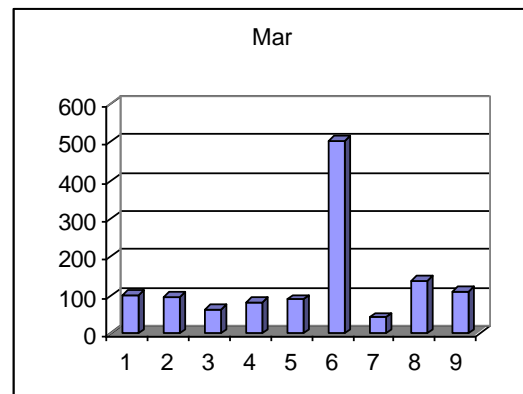
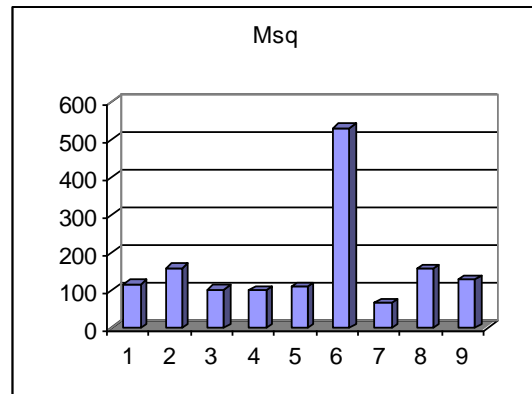
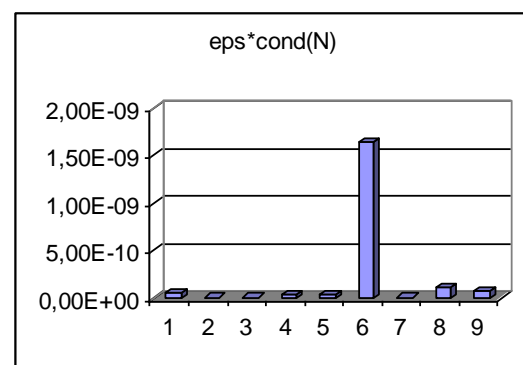
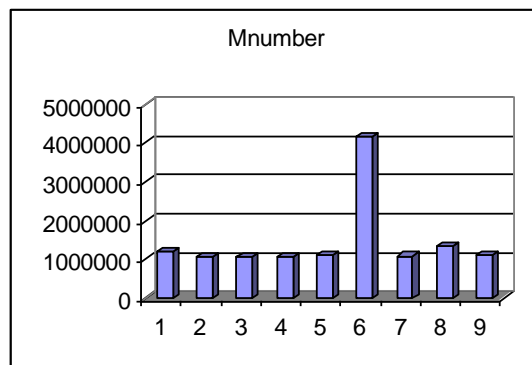
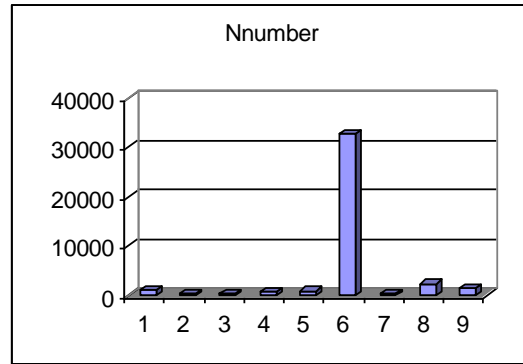
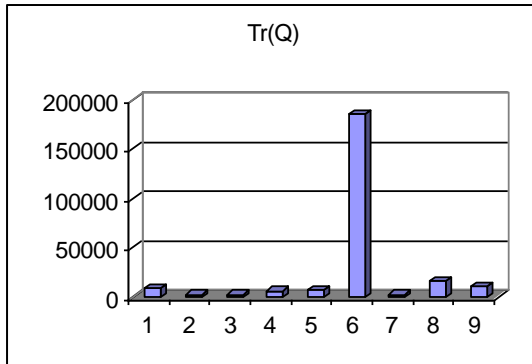


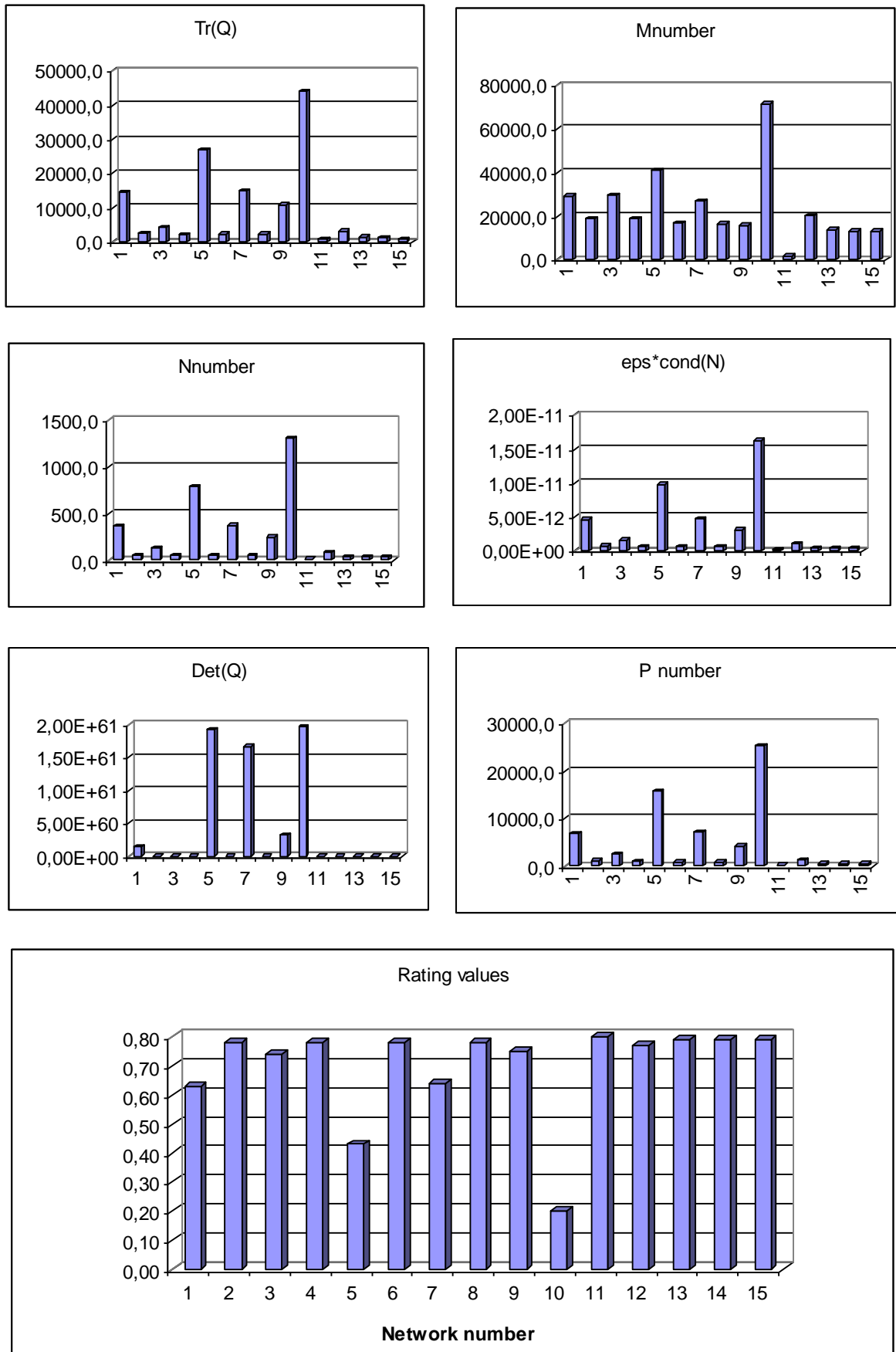
Table 3

Investigating the condition of a network with a square area shape,
fifteen variants

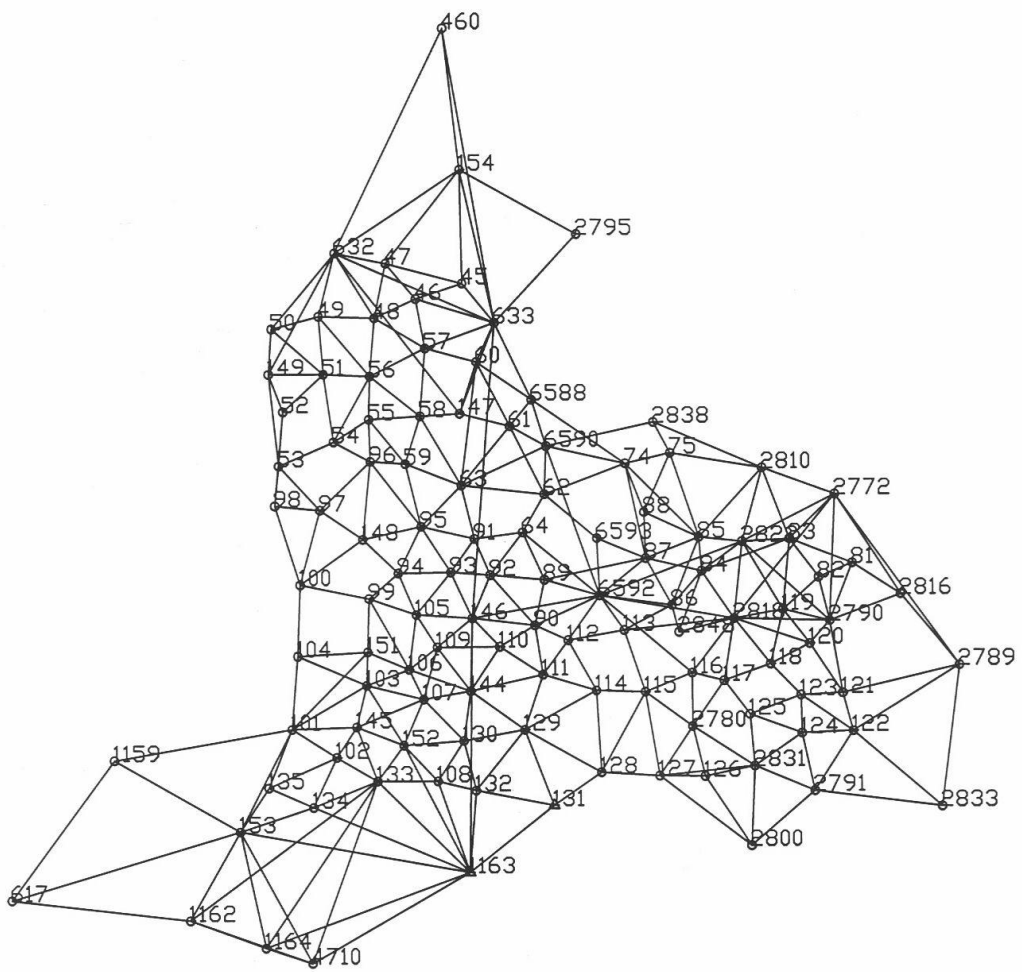
		Variables						
	Fixed points	$T_1(Q)$	$Mnumber$	$Nnumber$	$eps*cond(N)$	$Det(Q)$	$P number$	Rating
1	45, 47	14568,2	29409,2	369,8	4,60E-12	1,43E+60	6817,5	0,63
2	45, 47 and added coordinates I-st case	2586,8	18908,7	56,8	7,06E-13	3,43E+53	1066,2	0,78
3	45, 47 and added coordinates II-nd case	4248,9	29666,2	126,8	1,58E-12	2,41E+52	2493,9	0,74
4	45, 47 and added coordinates III-rd case	2234,6	18818,4	48,8	6,06E-13	7,94E+50	934,6	0,78
5	53, 98	26869,9	40976,6	787,3	9,79E-12	1,93E+61	15757,2	0,43
6	53, 98 and added coordinates	2344,9	16911,1	47,0	5,84E-13	3,58E+52	834,2	0,78
7	60, 633	14879,6	27033,9	375,0	4,66E-12	1,67E+61	7090,7	0,64
8	60, 633 and added coordinates	2358,2	16447,8	47,5	5,91E-13	1,48E+53	878,4	0,78
9	59, 95	10866,8	15983,2	246,4	3,06E-12	3,26E+60	4173,7	0,75
10	61, 6590	44028,1	71679,3	1306,2	1,62E-11	1,98E+61	25250,8	0,20
11	45, 47, 53, 59, 61, 95, 98, 6590	863,7	1783,8	12,2	1,19E-13	1,58E+40	159,7	0,80
12	45, 47 and added two coordinates	3194,0	20139,8	78,6	9,77E-13	6,54E+53	1317,4	0,77
13	45, 47 and added four coordinates	1554,8	13739,4	35,8	4,45E-13	2,05E+47	493,4	0,79
14	45, 47 and added six coordinates	1178,4	13184,4	32,8	4,07E-13	1,10E+41	460,2	0,79
15	45, 47 and added eight coordinates	917,6	13128,7	31,7	3,94E-13	4,58E+35	454,6	0,79

Figure 10

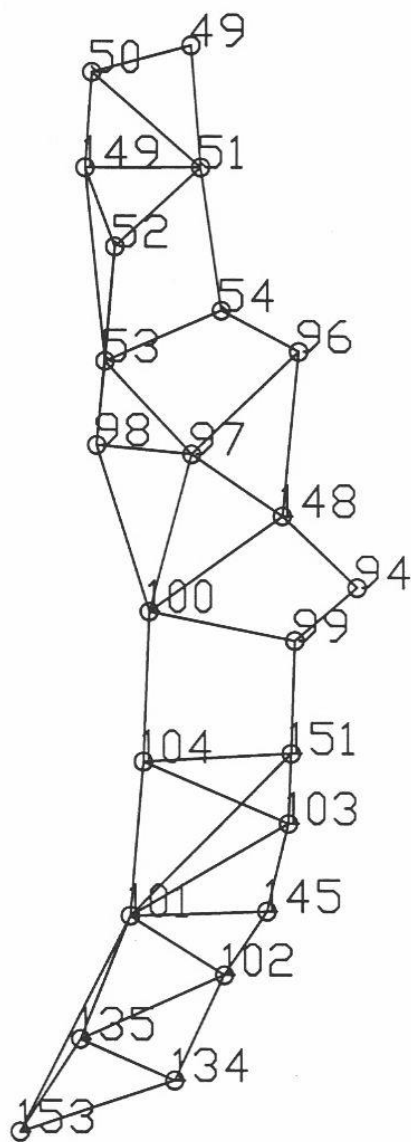
Values of the input variables and fuzzy logic rating of a network with a square area shape, fifteen variants



Plan of the network
Figure 4
M 1: 300 000



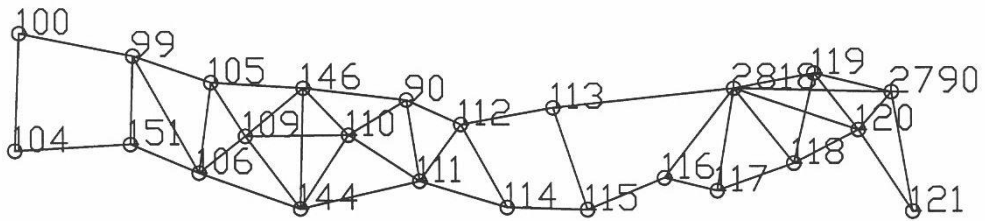
Plan of the network along the meridian
Figure 5
M 1:200 000



Plan of the network along the parallel

Figure 6

M 1: 200 000



Plan of the network with a square area shape

Figure 7

M 1: 200 000

